ANNOTATION

The thesis consists of 158 pages, which include in itself an introduction, three chapters with sections, conclusion and list of used sources.

The introduction of the dissertation contains a brief description of the current state of the research topic of the dissertation, the relevance and justification of the need for research. The first chapter first introduces well-known concepts and statements concerning the definition of a graph, the definition of spaces and operators, the associated Laplacian, discrete graphs and generalized Laplacians, metric graphs, quantum graphs and related operators associated with a discrete graph and to complete some simple consequences that needed later. For an arbitrary set of boundary conditions, an algorithm for constructing conjugate boundary forms is indicated. A complete description of all self-adjoint restrictions of the maximal operator is also given.

The second chapter examines a system of differential equations, which is a conjugate differential expression. Boundary conditions, boundary value problem, conjugate boundary value problem, regular boundary conditions, Green's function of an operator on an interval, perturbed boundary value problem, analytic nature of Green's function, differential operators on arbitrary connected geometric graphs without loops. is defined by differential expressions on arcs, Kirchhoff conditions at the inner vertices of the graph, for the introduced Lagrange formula. For an arbitrary set of boundary conditions, an algorithm for constructing conjugate boundary forms is indicated, and a complete description of all self-adjoint restrictions of the maximal operator is given.

In the third chapter, we consider the graph, the Green's function of the Dirichlet problem for a differential operator on a star-shaped graph, consisting of two infinite arcs and one arc of small length. This chapter discusses the basic concepts related to a graph-tree, the definition of a maximal operator on a graph-tree, the Lagrange formula for differential operators on a graph-tree, the Green's function of the Dirichlet problem for a differential operator on a graph-star for m, the d'Alembert formula in the case of nonsmooth multipoint problems for the wave equation, mixed problem for the wave equation.

In this thesis, it is proposed to consider a differential operator on a graph as an operator composed of differential operators on one-dimensional arcs and matrix operators on zero-dimensional inner vertices of the graph. Thus, a differential operator on a graph is a hybrid of differential and scalar matrix operators. The study of such operators from the point of view of spectral analysis is an urgent problem.

From a mathematical point of view, graphs are interesting because they are a good model for studying the properties of systems depending on the geometry and topology of space. Graphs are composed of zero-dimensional and one-dimensional manifolds, and in this sense seems interesting as a mixed dimension effect on those or other properties of mathematical objects defined on graphs.

From the practical point of view, first of all, when calculating elastic structures, it is necessary to study the problems of joining singularly degenerating regions. In this case, the structure can consist of areas of different dimensions. We need an asymptotic analysis of solutions to boundary value problems in domains that depend on sufficiently small dimensions of the domains and contract to the limiting "skeleton", which can be represented as a union of sets of different dimensions. Such objects are often called stratified sets.

In connection with the fact that the differential operators on graphs is not a "pure" differential, and are "hybrid" operators, so in the thesis put the goal: the mathematical apparatus, which is designed to study the spectral properties of the pure differential operators move to "hybrid" operators. Thus, the object of research is differential operators on graphs, as operators composed of scalar matrix and differential operators. To achieve this goal, the dissertation examines the following tasks:

- correctly define the maximum operator,

- derive the Lagrange formula for the maximum operator,

- construct the adjoint operator,

- describe all possible self-adjoint restrictions of the maximal operator,

- describe the correct restrictions of the maximal operator and the derivation of the formula for their resolvents,

- find the dependence of the resolvents on the lengths of the arcs of the graph and investigate the uniform resolvent convergence to the limit operator,

- indicate the spectral effects of the limit operator.

Provisions are taken out to protect: given a complete description of all possible self-adjoint operator of maximum constriction, discharged correct restriction maximum operator and found the formula their resolvent, installed dependency of the resolvents of the lengths of the arcs of the graph and investigated uniform resolvent convergence to limit operator given spectral effects limit operator.

The dissertation uses well- tested methods of the classical theory of ordinary differential equations, linear differential operators, methods of functional analysis, the apparatus of the spectral theory of unbounded operators, methods of the theory of functions of a complex variable, as well as methods of the theory of ODEs on graphs.

The novelty of the study is first installed oh dependence and resolvents of the lengths of the arcs of the graph, and illustrates the uniform resolvent convergence to the limit operator. Essential spectral effects of the limiting operator are indicated, which have not yet received due attention.

For a weighted discrete outer graph, we can interpret interior edges as edges of finite length and exterior edges as edges of infinite length with terminal vertices at "infinity". We will often use the following elementary fact about reordering a sum over edges and vertices. We also define the generalized boundary operator or the outer derivative associated with the vertex space. We use this outer derivative to define the associated Laplace operator.

This paper gives the basic concepts for the metric and quantum graphs and conclusions some general statements. Most of the material is standard and we refer to the literature for additional results and references. Since the metric graph is a topological space and isometric to intervals outside the vertices, we can introduce the concept of measurability and differentiate the function on the edges. If the boundary consists of all vertices, then we can give explicit formulas for the Dirichlet solution operator and the Dirichlet-Neumann map. It is necessary to Brother s note that in this case the boundary of the space boundary triple coincides with the whole space tops.

When writing the chapter, the main part of the Laurent series expansion of the Green's function of the perturbed operator in a neighborhood of a simple eigenvalue was calculated. In separate points, instead of general geometric graphs, we will consider their particular types: graph-tree and graph-star. Moreover, some of the above statements, such as the Lagrange formula and the description of self-adjoint contractions, will be much simpler. In particular, a Lagrange formula is given for a differential operator on a tree with Kirchhoff conditions at its interior vertices. The question of a complete description of correct restrictions of a given maximal differential operator on a tree shaped graph is studied. This subsection describes all self-adjoint restrictions of the maximal operator, as well as all invertible restrictions of the maximal operator. For this to start Give us known concepts related to the graph tree.

The concepts of trees and - an acyclic directed graph (directed graph without cycles), in which only one node has zero degree approach (do not keep the arc in it), and all other vertices have degree of call 1 (they lead exactly one arc). A vertex with a zero degree of entry is called the root of the tree, which we denote by 0, vertices with a zero degree of exit (from which no arc emanates) are called boundary vertices or leaves and are denoted by . Vertices that are not boundary are called internal. With read that the arcs have the same length. Next, that of a root out only one arc. Lagrange's formula plays an important role in the study of differential operators on an interval. An analogue of the Lagrange formula in the case of a differential operator on a tree graph is given in the paper. Also specify several lemmas.

In this paper, a system of second-order differential equations is investigated, which is a model of oscillatory systems with a bar structure. Problems for differential operators on graphs are currently being actively studied by mathematicians and have applications in quantum mechanics, organic chemistry, nanotechnology, waveguide theory, and other areas of natural science. The graph is a structure consisting of "abstract" segments and vertices, the adjoining of which is described by a certain relation. To define an operator on a given graph, it is necessary to select a set of boundary vertices. Vertices that are not boundary are called interior vertices. A differential operator on a given graph is determined not only by given differential expressions on arcs, but also by Kirchhoff-type conditions at the inner vertices of the graph.

In this paper, we have solved the Dirichlet problem for a differential operator on a star-shaped graph. We used the standard gluing conditions at the inner vertices and the Dirichlet boundary conditions at the boundary vertices. Also in this paper, the Green's function of a differential operator on a star graph is presented. Questions from spectral theory, such as the construction of the Green's function and the expansion in terms of eigenfunctions for models of connected rods, have been little studied. Spectral analysis of differential operators on graphs is the main mathematical tool for solving modern problems of quantum mechanics.

The main focus is on the spectrum of second-order differential operators on graphs. Various functional spaces on graphs. are defined, and we define, from the point of view of both differential systems and the aforementioned function spaces, boundary value problems on graphs. It is shown that a boundary value problem on a graph is spectrally equivalent to a system with a separated boundary condition. The main goal of this work is to solve the Dirichlet problem and construct his Green's function for a star-shaped graph. A star graph is a connected graph in which at most one vertex has degree greater than one. The vertex with degree greater than one is called the inner vertex of the star graph. Vertices that are not internal are called boundary vertices.

In this paper, we study the properties of the Green's functions of a boundary value problem for second-order differential equations on a star graph. In this subsection, we study the problem of the Fourier series expansion of the Green's function of the problem in terms of the eigenfunctions of the corresponding spectral problem. It is known that the solution to the Cauchy problem for the wave equation is given by the d'Alembert formula. The physical meaning of the d'Alembert formula corresponds to wave propagation. It is important that solutions to the wave equation can have discontinuities that propagate along the characteristics. Discontinuous solutions of the wave equation for a string and a rod have no physical meaning. However, the same equation is satisfied by the gas pressure in a long narrow pipe. The pressure can be bursting. Discontinuous solutions of the wave equation in gas dynamics are called shock waves. The d'Alembert method or the method of incident and reflected waves allows solving not only the Cauchy problem for the wave equation, but also finding solutions to mixed problems. In the case of a semi-bounded string, the effect of wave reflection is observed, which depends on the form of the boundary condition. In the case of bounded strings, waves are also reflected, but this effect occurs in a more complex scenario.

The papers in which the d'Alembert formula is modified for the mixed multipoint problem for the wave equation are considered. In this case, the solution to the mixed multipoint problem is assumed to be sufficiently smooth. Also considered operation where a is led to an analogue of formula Alembert mixed multipoint problem for the wave equation with initial data allow discontinuities of first derivatives. In this paper, we formulate and prove the d'Alembert formula for strings representing a graph-star.

Along with the indicated formulations of inverse problems for a set of spectra of reference problems, it is of interest to investigate the possibility of uniquely recovering only the boundary conditions of reference problems. We call such problems identification problems for boundary conditions. Sometimes this problem is called the problem of identifying the domain of the Sturm - Liouville operator, since the domain of the operator can be specified by different (but equivalent) sets of boundary conditions. The problems of identifying the boundary conditions of reference problems usually require an unambiguous reconstruction of a finite number of boundary coefficients.

In this paper, it is proved that for an unambiguous reconstruction of the boundary conditions, it is sufficient to specify only a finite number of eigenvalues ​​from each reference problem. A similar result was proved in the paper for higher-order differential operators on a finite interval. In this paper, a similar result is proved for the Sturm – Liouville operator on a star graph. In this case, special attention is paid to non-decaying boundary conditions. Boundary Damage Detection for Solid Connecting Structures. It remains a complex topic due to the influence of solids on each other and experimental conditions. Identifying boundary damage is difficult if the ends of the connecting structure are not accessible for visual inspection. Therefore, in this work, the natural frequencies of longitudinal vibrations of the connecting structures were used to identify boundary damage, since the natural vibration frequencies of the connecting structures can be measured by engineering sensors.

The subsections indicate a method for choosing reference problems, the spectra of which will make it possible to unambiguously find the boundary conditions of the original boundary value problem or equivalent boundary conditions. In fact, when determining the boundary coefficients, not the entire spectrum of the auxiliary reference problem is used, but only its finite part.

The validity and reliability of the scientific conclusions obtained in the dissertation are confirmed by their consistent theoretical and mathematical justification, as well as experimental data compared with technological and production data available in open sources.

Work wear t theoretical character. The results of work can find application in the future in the development of the spectral theory of boundary value problems on graphs and in the study of problems arising in the theory of elasticity, theory of stability and others. The reliability and validity of scientific positions, conclusions and results of the thesis is confirmed by publications produced results in journals with non-zero impact -factor.

All the results obtained are new and are based on our own solution methods. It was established the existence of a residue decomposition of all the functions in the area of determining the considered differential operator on the graph in a series of Fourier of own functions of the problem of Dirichlet.

Publications. The results of the dissertation were published in 13 papers. Of these, 3 articles in rating journals [40,41,42], 5 articles in journals recommended by the [43,44,45,46,47], 6 theses in the materials of international conferences [48,49,50,51, 52.53].